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Entropy Change in Axial Symmetric Gravitational Collapse

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We calculate the entropy changes of an imperfect-fluid gravitational collapsing system in the axial symmetric situation, analyzing a practical example. We conclude that the entropy of the collapsing system decreases at the beginning of the collapse and then increases very near the system's horizon.

1. INTRODUCTION

Many efforts have been made to explore the mechanism of the high entropy of black holes (Orkin *et al.*, 1981; Carmeli, 1982; Zurek and Page, 1984; Zhang *et al.*, 1986). Recently a new method has been used to analyze the problem (Yuan, 1994). It uses a distinguishing parameter for the entropy change. By using this parameter, in this paper we analyze a practical example, and obtain that the entropy of an axial symmetric gravitational collapsing system decreases at the beginning of collapse and does not increase until it is very near the horizon.

2. GENERAL EXPRESSION

We assume that the stress-energy tensor of an imperfect fluid is given by

$$T^{ab} = \rho g^{ab} + (p + \rho)\cdot u^a \cdot u^b + \Delta T^{ab} \tag{1}$$

$$\Delta T^{ab} = -\eta H^{ac}\cdot H^{bd}\cdot w_{cd} - \kappa(H^{ac}\cdot u^b + H^{bc}\cdot u^a)\cdot Q_c + \zeta H^{ab}\cdot (u^c)_{;c} \tag{2}$$

w_{ab} is the shear tensor:

$$w_{ab} = u_{a;b} + u_{b;a} - \tfrac{2}{3} g_{ab} u^c_{;c} \tag{3}$$

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Q_a is the thermal flux four-vector:

$$Q_a = T_{;a} + T \cdot u_{a;b} \cdot u^b \tag{4}$$

H_{ab} is the projection tensor:

$$H_{ab} = g_{ab} + u_a u_b \tag{5}$$

κ, η, and ζ are the thermal conductivity coefficient, shear viscosity coefficient, and volume viscosity coefficient, respectively.

We take the comoving metric to be

$$ds^2 = -(dx^0)^2 + A(dx^1)^2 + B(dx^2)^2 + C(dx^3)^2 \tag{6}$$

where A, B, and C are functions of x^a.

The distinguishing parameter of entropy change of the collapsing system is

$$\Delta = T^{-1} \cdot u_0 \cdot (\Delta T^{ab})_{;b} \tag{7}$$

The entropy increases when $\Delta > 0$, decreases when $\Delta < 0$, and is conserved when $\Delta = 0$.

For the cases $k = 0$ and $k \neq 0$, $\zeta = \eta = 0$, in a spherically symmetric system we showed that the entropies decrease (Yuan, 1994).

Here we consider an axial symmetric gravitational system as $k \neq 0$, $\zeta = \eta = 0$.

The dynamic axial symmetric method in slowing rotating is given by (Carmeli, 1982)

$$ds^2 = \left[1 - \frac{2mr}{\rho^2}\right] dv^2 + 2\, dv\, dr - \frac{4mra(t)\sin^2(\theta)}{\rho^2}\, dv\, d\varphi + \rho^2\, d^2\theta$$
$$- 2a(t)\sin^2(\theta)\, dr\, d\varphi + \left[r^2 + a^2(t) + \frac{2mra^2(t)\sin^2(\theta)}{\rho^2}\right] \sin^2(\theta)\, d\varphi^2 \tag{8a}$$

$$dv = dt + d\tilde{r} \tag{8b}$$

$$d\tilde{r} = \{[r^2 + a^2(t)]/\Delta_1\}\, dr \tag{8c}$$

$$\rho^2 = r^2 + a^2(t)\cos^2(\theta) \tag{8d}$$

$$\Delta_1 = r^2 + a^2(t) - 2mr \tag{8e}$$

Here we take m to be time independent, and a is very small. We can change the metric to a comoving metric as follows:

$$ds^2 = -dt^2 + dr^2 + \rho^2\, d\theta^2$$
$$+ \left\{r^2 + a^2(t)\sin^2(\theta) + \frac{[4mr - r^2 - a^2(t)\cos^2(\theta)]\sin^4(\theta)}{r^2 + a^2(t)\cos^2(\theta) - 2mr}\, a^2\right\} d^2\varphi \tag{9a}$$

$$A = r^2 + a^2(t)\sin^2(\theta) + \frac{4mr - r^2 - a^2(t)\cos^2(\theta)}{r^2 + a^2(t)\cos^2(\theta) - 2mr}\, a^2 \sin^4(\theta) \tag{9b}$$

$$B = 1 \tag{9c}$$

$$C = r^2 + a^2(t)\cos^2(\theta) \tag{9d}$$

Utilizing field equations and the distinguishing parameter, we neglect the terms higher than a, and then obtain

$$\Delta = \left\{ -2 - 3\sin^2(\theta) - 4r^2 + 5r^2\sin^2(\theta) - \frac{6mr\sin^2(\theta)}{(r-2m)^2} + \frac{4mr^2\sin^2(\theta)}{(r-2m)^3} \right.$$

$$\left. + \frac{1}{r-2m}\left[6m\sin^2(\theta) + 12mr^2\cos^2(\theta) - 6mr^2\sin^2(\theta)\right] \right\} \frac{a\dot{a}}{r^4 xT} \tag{10}$$

According to the relation (14) in Yuan (1994) we have

$$D_t S_T = \int \sqrt{-g}\Delta \, dx^1\, dx^2\, dx^3$$

$$= \int r^2 \sqrt{\frac{r}{r-2m}}\, \Delta \,|\sin(\theta)|\, d\varphi\, d\theta\, dr \tag{11}$$

where $\sqrt{-g} > 0$, $dx^1\, dx^2\, dx^3$, $r^4 xT > 0$, and usually $a\dot{a} > 0$. The sign of entropy change in a volume element with central mass m is only defined from the sign of

$$\Delta_a = -2 - 3\sin^2(\theta) - 4r^2 - 5r^2\sin^2(\theta) - \frac{6mr\sin^2(\theta)}{(r-2m)^2} + \frac{4mr^2\sin^2(\theta)}{(r-2m)^3}$$

$$+ \frac{1}{r-2m}\left[6m\sin^2(\theta) + 12mr^2\cos^2(\theta) - 6mr^2\sin(\theta)\right] \tag{12}$$

(θ, r) is the position of the volume element.

3. ANALYSIS

We can indicate the separate places where the entropy change is 0. When $\theta_0 < \theta < 90°$, $\Delta > 0$; when $0 < \theta < \theta_0$, $\Delta < 0$. When $180° > \theta > 90°$, things are the same as for $180° - \theta$, as in Fig. 1. This shows that when r is big enough, the angles approach 64° and when r is near its horizon, the various angles are quite different.

We can comment further about (11). Consider a general gravitational collapsing system. We suppose that a layer of collapsing matter encloses an

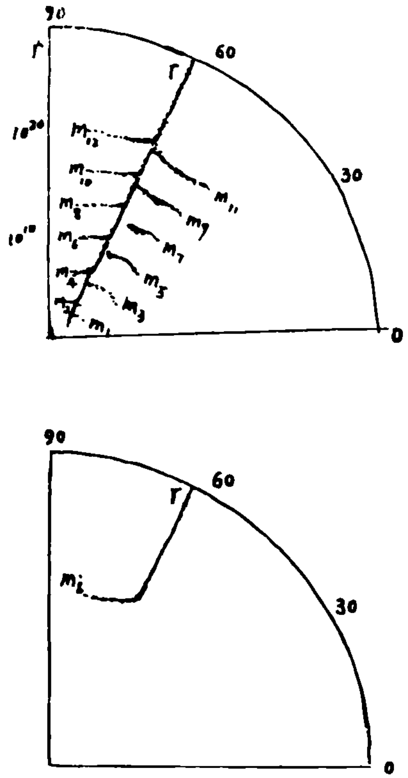

Fig. 1. Here $m_i = 10^{3i-18}M_\odot$, $i = 0, 1, \ldots, 12$.

axial symmetric mass m, and collapses from 10^{10} m to km. The whole entropy of a layer is

$$D_t S_T = \frac{8\pi a\dot{a}}{3xT}\left\{-10^{10}\,m - 5.7\times 10^{-10}\frac{1}{m} + \frac{1}{m}\left[3\sqrt{\frac{k-2}{k}} + 6\sqrt{\frac{k}{k-2}} - \frac{5}{3}\sqrt{\left(\frac{k}{k-2}\right)^3} + \frac{2}{5}\sqrt{\left(\frac{k}{k-2}\right)^5}\right] + m\sqrt{k(k-2)} + m\ln|1+k+\sqrt{k(k-2)}|\right\} \tag{13}$$

We plot the (m_0, k_0) on which $D_t S_T = 0$ in Fig. 2.

If m, k are larger than m_0, k_0, then $D_t S_T < 0$; if m, k are smaller than m_0, k_0, then $D_t S_T > 0$.

This shows that the entropy decreases at the beginning of the collapse process and does not increase until the layer is very near the horizon.

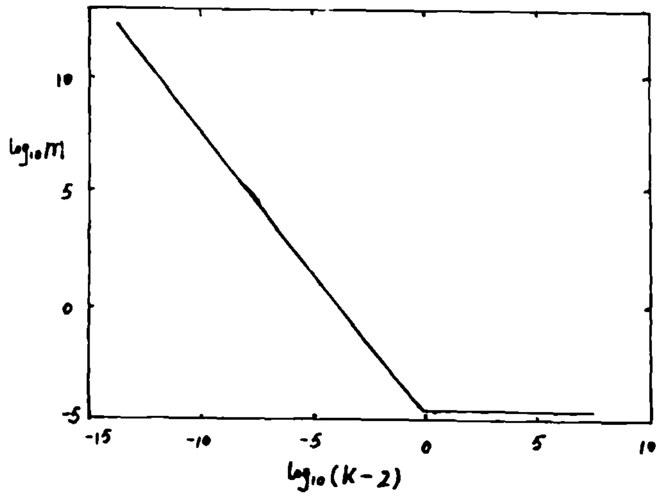

Fig. 2.

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